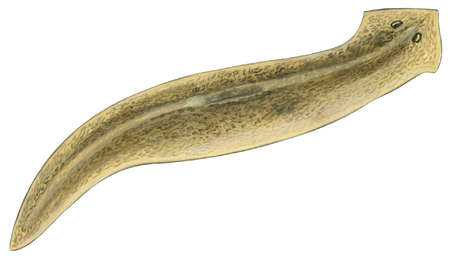
***Introduction to Theoretical Ecology Assignment 2***

Exponential Population Growth with Constant Immigration

1. You are a curious student in the Introduction to Theoretical Ecology course. After the class, you decide to do a small experiment on population growth. You set up a “massive” fish tank and introduce *N0*flatworm individuals. Also, each day you add *I* new individuals into the tank, hoping that the population will increase faster. Assuming that the intrinsic rate of increase is *r* (per day) and there is no factor limiting the growth and reproduction of these flatworms, the population dynamics can be described by the following differential equation:

The analytical solution to this differential equation is:

Please use what you have learned in the lecture to derive the solution from the differential equation step by step. (You can either write down the answer on a paper and embed a picture of it or directly type the equations in Word.)



***Solution:***

*Step 1. Rewrite RHS of the equation:*

*Step 2. Rearrange the terms to separate the variables:*

*Step 3. Integrate both sides: ,*

*Step 4. Take exponential of both sides:*

*Step 5. Plug in the initial values:*

*Step 6. Substitute C2 into the equation in step 4:*

*Step 7. Rearrange:*

1. Suppose that *N0* = 10, *r* = 1.2, and *I* = 3, how will the flatworm population change over a week? Solve the differential equation numerically and visualize the population trajectory. Please show the figure along with the R code you use to generate the results. (You can use any R graphic system you like for plotting).

***Solution:***

**plot.tiff**

**R code:**

**library(deSolve)**

**library(tidyverse)**

**exponential\_model <- function(times, state, parms) {**

**with(as.list(c(state, parms)), {**

**dN\_dt = r\*N + I**

**return(list(c(dN\_dt))) })**

**}**

**times <- seq(0, 7, by = 0.1)**

**state <- c(N = 10)**

**parms <- c(r = 1.2, I = 3)**

**pop\_size <- ode(func = exponential\_model, times = times, y = state, parms = parms)**

**ggplot(data = as.data.frame(pop\_size), aes(x = time, y = N)) +**

**geom\_point() +**

**labs(title = paste0("Exponential Growth \n (r = ", parms["r"], ")")) +**

**theme\_classic(base\_size = 12) +**

**theme(plot.title = element\_text(hjust = 0.5)) +**

**scale\_x\_continuous(limits = c(0,7.5), expand = c(0, 0)) +**

**scale\_y\_continuous(limits = c(0, max(as.data.frame(pop\_size)$N)\*1.1), expand = c(0, 0))**

1. Compare the population growth with and without constant immigration and explain the model dynamics in your own words. What is *I* doing? Do you think your daily addition of new flatworm individuals make a big difference?

***Solution:***

Population growth with *I:*

Population growth without *I:*

As you can see, the population growth with *I* is actually the population growth without *I* plus another exponential term. So adding a constant *I* to the original differential equation will in fact lead to an exponential addition of individuals.

plot2.tiff

plot4.tiff

Also, the ratio of the population size with *I* to the population size without *I* increases in the beginning and level offs over time.

plot3.tiff